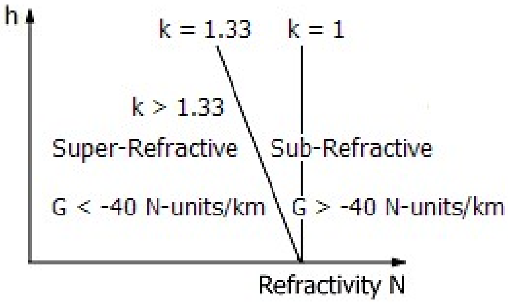
Training Program : Point-to-Point Radio Link Design
Course : Multipath Fading
COMPLEMENTS to Lesson 1 : Propagation in Stratified Atmosphere

Refractivity profiles

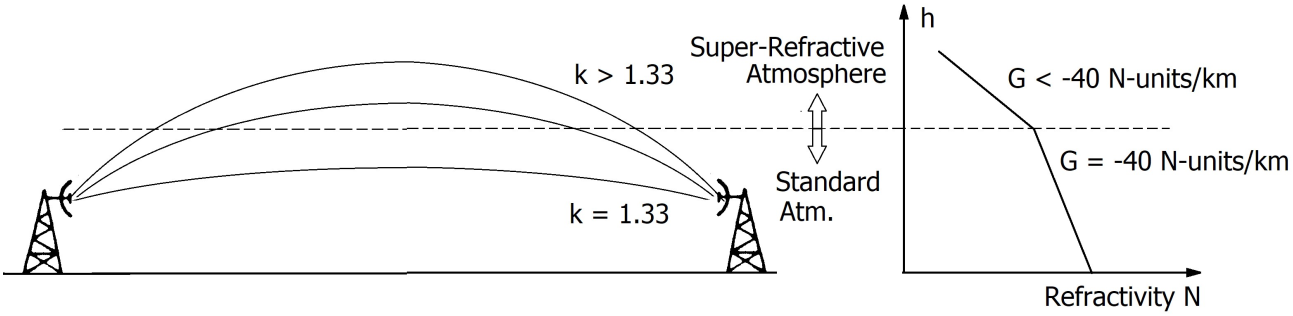
A review of Refractivity profiles is useful to introduce the concept of stratified atmosphere and the effects on EM propagation.

In the figure, the Sub-Refractive and Super-Refractive regions are shown, corresponding to k values below and above 1.33, respectively.

The k=1 case indicates the condition of homogeneous atmosphere (G=0), when the wavefront moves along a straight line and the Equivalent Earth radius is equal to the Real Earth radius.



Below, a simplified representation of stratified atmosphere is shown, which may generate multipath propagation conditions.



In addition to Refractivity N and Refractivity Gradient G, a Modified Refractivity M is defined as:

$$M = N + (h/R) \cdot 10^6$$

where h is the altitude a.s.l. and R is the earth radius (h and R expressed in the same unit). With R = 6370 km, we have:

$$M = N + h[\mathrm{km}] \cdot 157 \qquad dM/dh = dN/dh + 157 = G + 157 \ \ [\text{M-units / km}]$$

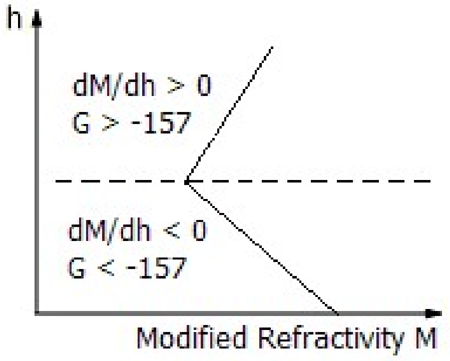
Modified Refractivity make easier to identify extreme stratification conditions, producing Duct formation, with possible black-out of the received signal.

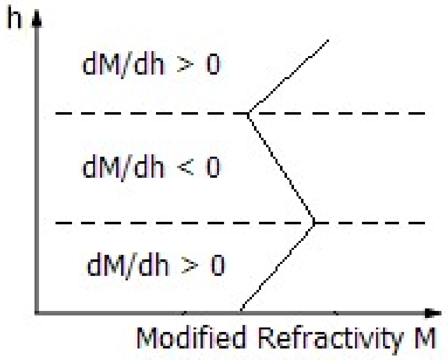
From the dM/dh formula, it follows that if $dM/dh = 0$, then $G = -157$ and $k = \infty$; this means a ray curvature equal to the earth curvature (ray trajectory parallel to the earth surface). For $dM/dh < 0$, then $G < -157$ and the ray trajectory bends toward the ground. On the other hand, for $dM/dh > 0$, then $G > -157$ and the ray trajectory moves away from the ground.

The two figures below show two examples of atmosphere stratification with anomalous M profiles, thus producing duct conditions [1].

In the first case, a ground-based layer is formed, where part of the emitted wavefront is bent downward to ground, while above that layer the wavefront is bent upward. Depending on the positions of the Tx and Rx antenna, a blackout condition may occur.

Similarly, in the second figure, an elevated duct is formed in the region with $dM/dh < 0$.





Random Vectors and Rayleigh Distribution

Let us consider a collection of vectors (in exponential notation) $V_i = A_i \exp(j \cdot \phi_i)$ where A_i and ϕ_i are random variables and ϕ_i is uniformly distributed in the range $-\pi$ --- $+\pi$. In orthogonal coordinates, the two V_i components are:

$$x_i = A_i \cos(\phi_i) \quad \text{and} \quad y_i = A_i \sin(\phi_i)$$

When N vectors V_i are added, then we get the vector V_N

$$V_N = A_N \exp(j \cdot \phi_N) = \sum_i V_i = x_N + j \cdot y_N$$

with $x_N = A_N \cos(\phi_N) = \sum_i x_i$ $y_N = A_N \sin(\phi_N) = \sum_i y_i$ and modulus $A_N = \sqrt{(x_N^2 + y_N^2)}$

Many contributions can be found in the literature which investigate on the statistical distribution of A_N. In particular, let us consider the case when N --> ∞. While a complete presentation is given (among others) by Beckmann (see references), we try to summarize the basic steps to get the final result.

Under constraints applicable to the variance of the A_i random variables (which are satisfied in practical cases), it can be proved that the Central Limit Theorem can be applied [2].

As a result, x_N and y_N (N --> ∞) are independent Random Variables with Gaussian distribution. This proves that A_N (N --> ∞) is a Random Variable with Rayleigh distribution.

Useful References

[1] Seydol J.S., "Introduction to RF Propagation", Wiley, 2005

[2] P. Beckmann, "Rayleigh Distribution and Its Generalizations", RADIO SCIENCE Journal of Research NBS/USNC-URSI, Vol. 68D, No.9, 1964