Training Program : Point-to-Point Radio Link Design

Course : Path Analysis, Obstructions

COMPLEMENTS to Lesson 2 : The k-factor and Fresnel ellipsoid

Antenna pointing, launching angle

If we assume a constant vertical Refractivity Gradient, then the ray trajectory suffers the same curvature, at any elevation in the atmosphere, and a direct ray trajectory is identified, from the Tx antenna to the Rx antenna. The launching angle α (at the transmitter site) is computed as :

$$\alpha = -\,\mathrm{arctg}\,\{ (H_T - H_R) / \beta + [\beta / (2 \cdot R_E)] \}$$

where $\beta = -\sqrt{[D^2 - (H_T - H_R)^2]}$

R_E is the equivalent earth radius (8500 km with standard k-factor = 1.33)

$H_T - H_R$ (in m) is the difference in antenna elevations a.s.l.

D is the path length (in km).

With a limited accuracy, an estimate of the k-factor can be computed once a radio hop is installed and the Tx antenna has been pointed. In that case, α corresponds to the antenna elevation angle and the above formula can be inverted. This leads to compute the equivalent earth radius $R_E = k \cdot 6370$ and an estimate of the k-factor.

Minimum k-factor in Tropical climate

Along the radio path, the Refractivity Gradient in the atmosphere may vary, so that point values and Gradient statistics in a single location are not significant when dealing with medium-long radio hops. As an example, horizontal distribution of Refractivity Gradient is studied in [1].

This leads to the definition of an "Effective k-factor k_E", as a function of hop length. The minimum k_E value in "continental temperate climate" is shown in Fig. 2 of ITU-R

Rec. P.530 [2]. It is useful to look for additional information about prediction of the k_E value in tropical climate, where we expect large variability in the refractivity gradient.

In [3] a method is presented to derive the minimum k_E from climatological refractive index statistics available for many locations worldwide.

A tropical site (Dakar, Senegal) gives an example of results. They can be approximated, for distances in the range 10 to 100 km, using the formula:

$$\text{Min } k_E = 0.15 \cdot [\text{Log}_{10} D]^2 + 0.23$$

In comparison with ITU-R (temperate climate) data, the above results show a reduction of the minimum k_E values of the order of 15 - 20%..

It should be noted that this example refers to a specific tropical site. Given the wide variability of refractivity data in tropical areas, it cannot be considered as fully representative of that region.

Radio propagation and visual analogies

We are familiar with our visual experience and this can be of help in describing some aspects of radio propagation.

However, the Fresnel ellipsoid shows that radio propagation (like any EM propagation effect) cannot be explained only in terms of geometric optics, that is adequate so long as any discontinuities encountered through the propagation path are very large compared with the wavelength.

The ellipsoid radius is proportional to the wavelength square root. In our visual experience, the light wavelength is so small (about 5 10^{-4} mm) that the radius of the Fresnel ellipsoid is negligible, at least as a first approximation. Diffraction effects can be observed only with accurate experiments, showing the role of Fresnel ellipsoid also in the optical field.

On the other hand, in radio communications the wavelength is in the range from 1 m (frequency 300 MHz) to about 1 cm (frequency 30 GHz), that is almost one million times larger then in visible waves.

In conclusion, much care must be paid in establishing an analogy between radio propagation and visual experience. Even if in both cases we deal with EM waves, the large difference in wavelength makes practical results quite different in most conditions. For example, the concept of Visibility is quite different in EM wave propagation and in our visual experience.

Useful References

[1] Grabner M. et al., "On horizontal distribution of vertical gradient of atmospheric refractivity", Atm. Sci. Let. 18 (294-299) (2017)

[2] ITU-R Rec. P.530-18, "Propagation data and prediction methods required for the design of terrestrial line-of-sight systems", 2021

[3] Mojoli L.F., "A new approach to visibility problems in line-of-sight hops", National Telecomm. Conf., Washington, 1979