



Training Program : Point-to-Point Radio Link Design

Course : Path Analysis, Obstructions

COMPLEMENTS to Lesson 1 : Refractivity in the Atmosphere

## Refractivity formulas

The atmospheric radio refractive index "n" is defined as:

$$n = \frac{\text{Propagation speed in the vacuum}}{\text{Propagation speed in the medium}}$$

Since n is very close to 1, it is convenient to define a related index N (Atmosphere Refractivity) as [1, 2]:

$$N = (n - 1) \cdot 10^6$$

Both n and N are dimensionless indexes, N is measured in [N-units].

Atmosphere Refractivity can be computed as a function of atmospheric pressure **P** [hPa], water vapor pressure **e** [hPa] and absolute temperature **T** [K] as :

$$N = 77.6 \frac{P}{T} - 5.6 \frac{e}{T} + 3.75 \times 10^5 \frac{e}{T^2}$$

An approximate expression for N is :

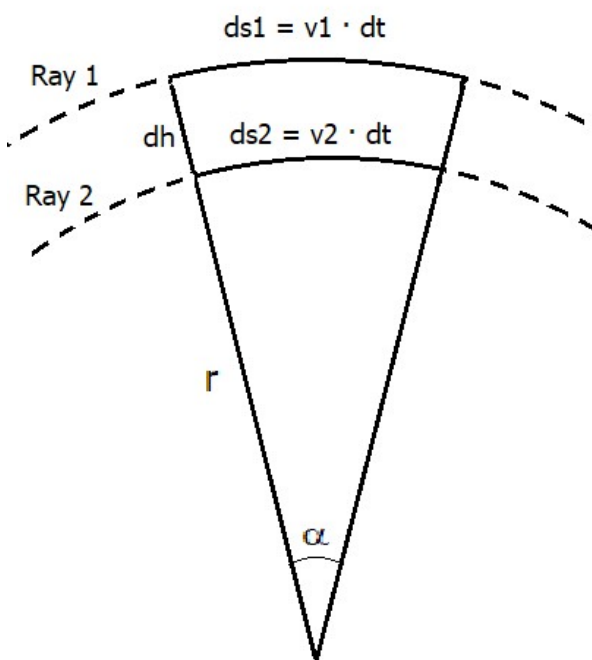
$$N = \frac{77.6}{T} \left( P + 4810 \frac{e}{T} \right)$$

Refractivity Gradient G is defined as  $G = dN / dh$  and is measured in [N-units / km].  
Refractivity definition gives also:

$$G = (dn / dh) 10^6$$

## Curvature of EM ray

The curvature of the EM wavefront can be computed as a function of local Refractivity Gradient  $G$ , measured in [N-units/km]. If a constant  $G$  is assumed in all the space interested by EM propagation, then the same curvature applies to the whole ray trajectory.



With reference to the Figure, two rays in the wavefront are considered,  $r_1$  and  $r_2$ . Vertical spacing between the two rays is equal to  $dh$ , assuming a low elevation angle.

In the time  $dt$ , the two rays will travel the distance  $ds_1$  and  $ds_2$ , with speed  $v_1$  and  $v_2$ , respectively, following a circle arc.

The following relations hold :

$$ds_1 / ds_2 = v_1 / v_2 = n_2 / n_1 = n / (n + dn)$$

(speed is inversely proportional to refractive index  $n$ )

$$ds_1 / ds_2 = (r + dh) / r = 1 + (dh / r)$$

(both arcs insist on the same angle  $\alpha$ )

As a result from the two previous formulas (both giving  $ds_1 / ds_2$ ), we get :

$1 + (dh / r) = n / (n + dn) = 1 - dn / n$        $dh / r = - dn / n$  Taking account that the refraction index  $n$  is very close to 1, so that  $1/n \sim 1$ , the ray curvature  $1/r$  is finally given by :

$$1/r = - 1/n \cdot (dn / dh) \sim - (dn / dh) = - G \cdot 10^{-6}$$

## Useful References

[1] [ITU-R Rec. P.526-15, "Propagation by Diffraction", 2019](#)

[2] [Seydol J.S., "Introduction to RF Propagation", Wiley, 2005](#)